WHAT STUDNETS DO WHEN HEARING OTHERS EXPLAINING

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The purpose of this paper is to investigate what students do when hearing others' explanations. For this purpose, we set two video-cameras in a classroom, one of which recorded only the target student throughout the lesson and one of which recorded the teacher and other students explaining at the blackboard. The learning processes of two elementary school students will be analyzed. The analysis will demonstrate that the students induced their own learning by putting what they had noticed into new contexts activated by the information which they had selected from the others' explanations.

INTRODUCTION

It is widely recognized today that discussions in classrooms are important for learning mathematics. To help students participate in such discussions effectively and, as a consequence, construct mathematical knowledge, many studies have investigated discussions in whole-class or small-group settings and deepened our understanding of social aspects of mathematics classes. For example, Yackel (1995) analyzed the discussions in the inquiry mathematics classrooms and illuminated some factors which can influence students' explanations or constitute a situation for explanation. Webb et al. (2002) analyzed small-group works and pointed out that help-seekers needed to ask appropriate questions or requests to elicit good explanations.

In Japan, mathematics lessons, especially introductions of new ideas, include the time of discussing solution methods after students work individually or with their neighbors (see Stigler & Hiebert, 1999, p. 79). In this discussion, some students explain their ideas to the class. Even though they explain their ideas and some of other students ask questions about their explanations, many of the classmates spend most of the time hearing others explaining or asking. If so, we should pay attention to students who hear others explaining, as well as those who explain their ideas.

While the importance of social aspects has gotten our attention, some researchers seem to direct our attention to individual students who learn mathematics in social settings like classrooms. For example, Waschescio's (1998) indication, which was made after critically reviewing some researches about social constructivism in mathematics education, seems to imply that we should pay more attention to manners of learning of individual students. Anthony (1996) and Nagashima (1998) analyzed learning processes of individual students in ordinary secondary mathematics lessons and illuminated the different learning goals they had.

Following such indications, it may be valuable to investigate what individual students do or experience when they hear others explaining in mathematics classrooms. The purpose of this paper is to analyze the learning processes of students, focusing on the phase where they hear others' explanations, in order to gain insights into what kinds of learning can occur in such phases.

GATHERING DATA

In the following sections, we will analyze the learning processes of two students: One is a fifth grade male student, Shingo, and another is a sixth grade female student, Shizue. They attended different elementary schools in Japan. To investigate what students do when they hear others' explanations, in observing each lesson, we set a video camera so that it could record only the targeted student throughout the lesson. We sometimes operated the camera to zoom in and record how the student worked on his/her worksheet. Another video camera was set at the back of the classroom. It recorded the teacher and other students explaining at the blackboard. We made the transcripts from the videotaped records. They included what the target students did and what happened in the classroom (e.g. what kinds of explanation were presented by other students) at that time. These transcripts and the videotaped records are the data used in the following analyses. All the names mentioned in this paper are pseudonyms.

EPISODE1: SHINGO'S LEARNING

Shingo's behaviors during working individually

In his 5th grade class, the topic was to expressing the quotient of two whole numbers with fractions. The teacher posed the following task to the class: "We want to divide 2ℓ of milk among three people. How many liters of milk can one person get?"

Shingo wrote "2÷3" and started its long division algorithm. As it continued endlessly, he changed "2÷3" into "3÷2" after looking at the neighbor's notebook. At this moment, in responding to other students' question, the teacher announced that "3÷2" did not fit the situation of the task. He returned to "2÷3." The teacher interrupted and initiated the whole-class discussion. When some students mentioned that 2 was not divisible by 3, Shingo implemented a long division of 2÷3 and rounded off its quotient to obtain 0.67 and 0.7. He also calculated 0.67×3 and 0.7×3. After that, some students mentioned the rounded-off answer 0.7 and the class confirmed that it was an approximate value. The teacher encouraged the student to devise the way of expressing the quantity accurately.

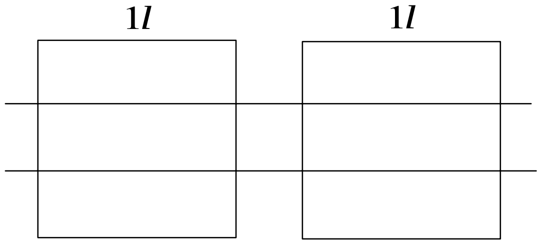

Fig. 1

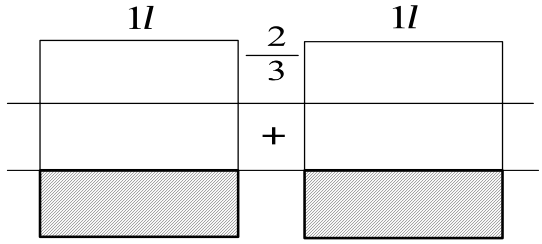

Fig. 2

The teacher drew two squares on the blackboard and added two horizontal lines crossing these squares. When he said, "I've divided it into three parts. Do you understand?” Shingo nodded slightly. The teacher handed the students worksheets which included the diagram shown in Fig. 1. He asked them to paint the area representing the quantity for one person, express it with a fraction, and write the reason for using that fraction. Shingo painted the lowest two rectangles (Fig. 2: There were not “2/3” and “+” at this moment). Hearing someone saying “two thirds,” Shingo wrote down “2/3” on his worksheet. But he erased this immediately.

After that, Shingo wrote his explanation on the worksheet: “When dividing 2ℓ among three people, a share for one person is 1/3. And two pieces of that.” He looked at his worksheet for about 30 seconds and modified and extended this explanation as follows: “When dividing 1ℓ among three people, a share for one person is 1/3. If changing it to 2ℓ, I can paint two pieces for one person. So, I can paint one piece for 1ℓ.” During Shingo was writing the last sentence, the teacher initiated the whole-class discussion again. He worked on his worksheet for about 15 minutes, aside from 6 minutes for the intervening whole-class discussion.

What did he do when hearing others’ explanations

The whole-class discussion started with the explanation of two students who thought the quantity for one person to be 2/6. They told that it was 2/6 because two squares in Fig. 1 were divided into six equal pieces and two pieces of them were a share for one person. Shingo looked at his worksheet as he listened to the explanation of the second student. Following these students’ explanation, the teacher said, “The expression 2÷3 becomes two sixth, doesn’t it?” Shingo nodded a few times.

At this moment, another student, Masato, raised his hand and insisted that the quantity for one person was two thirds. He went to the blackboard and explained his idea as follows: (i) Dividing the left square, which representing 1ℓ, into three pieces; (ii) Adding 1/3 from them and another 1/3 from the right square; (iii) Since adding them led to 2/3, 2÷3 became 2/3. The teacher repeated his explanation with Masato. When the teacher confirmed with Masato whether two 1/3’s were added, Shingo wrote down “+” and “2/3” between two squares in his worksheet (see Fig. 2). Some students uttered that they could understand the both ideas. The teacher told to the class that they thought there were two answers. Shingo put his head on one side.

Four students expressed their opinions: (Ken’ya) since two squares are separated, the answer is 2/6; (Ikumi) the answer must be one of 2/6 and 2/3, but the opinion will be divided; (Yuko) 2/3 resembles the original expression 2÷3; (Masato) as 2/6 means that 6 people share the milk, 2/3 may be better. When Masato expressed his opinion, Shingo told to the neighbor girl as follows: “It is 2/3. ‘Cause, speaking in terms of fractions, 1/3, 2/3…it is not 2/6.” After that, the teacher asked the class whether there were two answers to 2÷3. While one student spoke loud that he was not sure, Shingo told to the neighbor girl as follows: “But 1/3 and 1/3 does not become 2/6.”

When the teacher encouraged the class to resolve this question by themselves, Shingo raised his hand and told to the class as follows: “Adding 1/3 and 1/3 usually becomes 2/3. The denominator does not change in such addition.” Another boy, Sou, mentioned referring to the worksheet used in the previous lesson that 2/6 was not equivalent to 2/3 and it was less than 2/3. The teacher confirmed with the class that 2/6 was equivalent to 1/3 and 3/9. During this discussion, Shingo looked at the teacher and speaking children, but he wrote or spoke nothing. Finally, Ikumi said that she could understand both ideas and the teacher announced to keep thinking about this issue in the next lesson.

Discussion about Shingo's learning

Even at the end of individual workings, he did not write how many liters of milk one could get. He only wrote that two pieces should be painted for one person. In the first half of the whole-class discussion, he nodded a few times when the teacher mentioned 2/6 as the answer. Shingo did not know 2÷3=2/3 at all when the whole-class discussion began. In the second half of the whole-class discussion, Shingo raised his hand and set out his idea which supported the opinion that the answer was 2/3. When the teacher made an announcement about the next lesson, Shingo said to the neighbor girl, "It is two thirds, since the denominator does not change." Shingo had arrived at the conviction that 2÷3 becomes 2/3 through the whole-class discussion of this lesson.

Because Shingo spontaneously wrote the answer "2/3" on his worksheet when hearing Masato's first explanation, this explanation can be considered critical for Shingo to understand 2÷3=2/3. In fact, when the teacher mentioned the existence of two answers immediately after this explanation, Shingo put his head on one side. When other students expressed their various opinions, Shingo told the neighbor girl a few times that the answer was not 2/6 but 2/3. His behavior changed before and after the Masato's first explanation.

In repeating the Masato's explanation, the teacher wrote "1/3" in two painted pieces of the diagram on the blackboard. But Shingo did not write these "1/3" on his worksheet. On the other hand, he wrote down the "+" sign on his worksheet before the teacher used this sign to write "1/3+1/3=2/3." In the Masato's explanation, the idea of addition was most important for Shingo.

He might attend to this idea because it could bridge the gap between what he had done during his individual working and the goal of expressing the quantity with a fraction. Through his individual working, Shingo had realized that when dividing 1ℓ of milk among three people, the quantity for one person became 1/3. He had also found that when dividing 2ℓ, the quantity for one person could be expressed by two out of six pieces. The Masato's explanation made it possible for Shingo to put these findings in the context of addition of fractions. Viewing them in this context, Shingo could integrate his findings to achieve the above goal. Furthermore, in this context, he could bring forward another reason in favor of the answer 2/3: "The denominator does not change in such addition." No students mentioned this reason before he presented it.

Before hearing the Masato's explanation, Shingo might view these two pieces in the context of the number of equally-divided pieces. This is a reason why he nodded a few times when the teacher mentioned the answer 2/6, although he noticed that each piece represented 1/3. What he knew about the problem situation was basically the same before and after the Masato's explanation. Through his peer's explanation, however, Shingo could put it in another context and arrive at a certain conviction.

EPISODE 2: SHIZUE'S LEARNING

Shizue's behaviors during working individually

In her sixth-grade class, the topic was an introduction of division of fractions. The students worked on the following task using the worksheet (Fig. 3): "3/4$d\ell$ of paint is needed to paint 2/5m^2 of wall. How many square-meters of wall can be painted using 1$d\ell$ of paint?" Before working individually, the class reflected the preceding lesson and made sure that the expression for this task was 2/5÷3/4. Since they had not yet learnt how to calculate it, the teacher asked the students to devise and find out the area.

Shizue painted 3/4 of the 2/5 m^2 wall (Fig. 4) and wrote "2/5÷3/4=8/15". She erased this "8/15" and the painted part, and added two bars as shown in Fig. 5. Shizue moved her pencil in the air over "2/5÷3/4" for a while and wrote "8/15" again. She painted the part expressing 2/5 m^2 wall (i.e. below two rows in the square). She seemed to get in a bind. Then she made "the reduction" of "2" and "4" and changed the answer to 2/15.

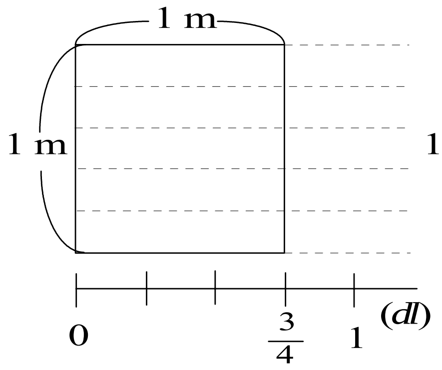

Fig. 3

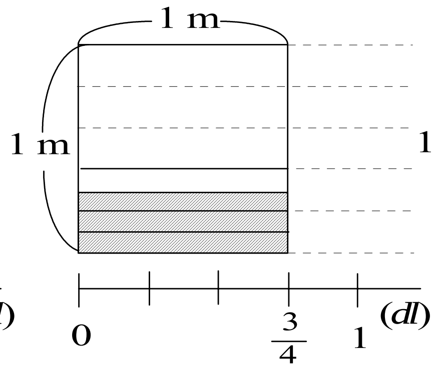

Fig. 4

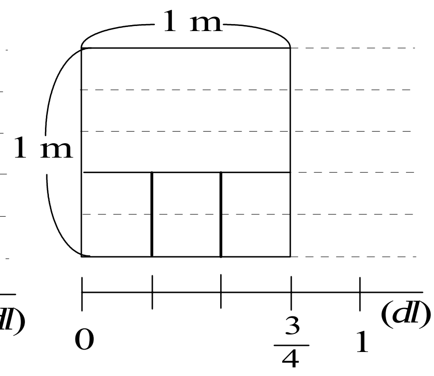

Fig. 5

When the teacher handed her a hint card, she erased this expression. This card included the diagram shown in Fig. 6 and its hint was to mark the area which could be painted by 1$d\ell$ of paint. She added "1/4" and "2/4" to the number line and counted 6 small

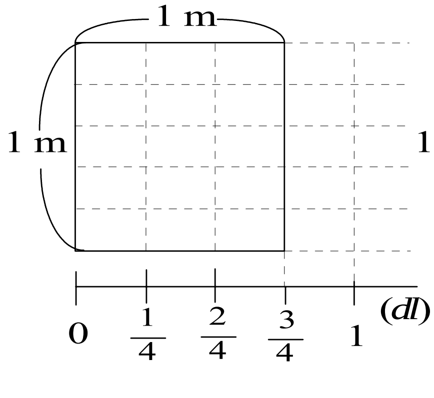

Fig. 6

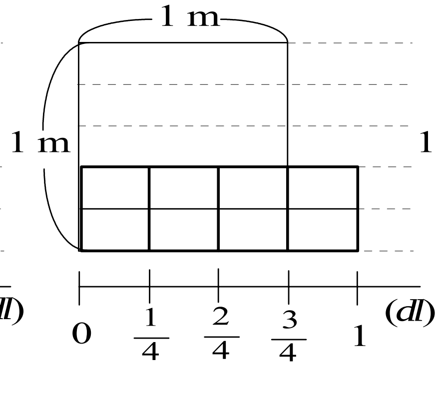

Fig. 7

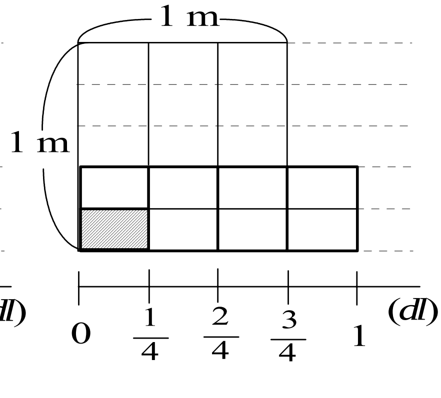

Fig. 8

rectangles in Fig. 5. After that, Shizue drew lines enclosing that area (i.e. a larger rectangle) in the worksheet (Fig. 7). She quickly counted 8 small rectangles in this area one by one. Then she wrote the expression "3/4÷2/5= /2" with "the reduction" of "4" and "2." However, she erased this expression at once.

The teacher handed her another hint card. It asked how many square-meters the leftmost two rectangles were (i.e. the area which can be painted by 1/4 $d\ell$) and asked how many pieces of such part constituted the area painted by $1d\ell$ of paint. Shizue wrote "1/4" as the answer to the first question and "4/4=1" as the answer to the second question. After that, the teacher called on one of her classmates, Nana, to share her idea with the class. Shizue's individual working lasted about 23 minutes.

What Shizue did during hearing others' explanations

The teacher copied Nana's diagram (Fig. 8) on the blackboard. Because Nana hesitated to explain her idea by herself, the teacher explained it to the class using that diagram and reading her worksheet. Nana's idea was as follows: (i) In Fig. 8, the area which can be painted by $1d\ell$ of paint is the big rectangle enclosed by bold lines; (ii) Draw vertical lines at 1/4 and 2/4; (iii) There are 15 small rectangles in the $1m^2$ wall represented by a square; (iv) Since the area painted by $1d\ell$ of paint is consisted of 8 small rectangles, it has an area of 8/15 m^2.

After the teacher explained the step (i), Shizue wrote "2/5÷4/4=2/20" ("2" of 2/5 was cancelled by the denominator "4" of 4/4). However, when the teacher explained the step (ii), Shizue erased this expression and looked at the blackboard again. The teacher proceeded to the step (iii) and counted 15 small rectangles one by one. When he counted the 13th rectangle, Shizue looked at her worksheet and added two lines to her diagram (Fig. 7) to change it into one like Fig. 8. Then she counted 15 small rectangles one by one. She also pointed the two rightmost rectangles in the bold lines. After she heard the teacher's explanation of the step (iv), Shizue wrote "2/5÷3/4=8/15" on her worksheet. Though another student began to explain her idea, which was different from Nana's one, Shizue erased this expression and attempted to write down the step (iii) with her own words. After several such attempts, Shizue wrote down on her worksheet what the teacher wrote on the blackboard. When she finished writing it down, the third student explained his idea based on a number line. She wrote nothing on her worksheet concerning the ideas of the second and third students.

Discussion about Shizue's learning

According to the post-interview, Shizue obtained "2/5÷3/4=8/15" by multiplying numerators and denominators crossly. She might learn this method outside school. Although she implemented this calculation at the beginning of her individual working, she did not seem to be certain of how to use this method. She modified the above expression into one with the "reduction" and changed its answer into 2/15. She erased this modified version soon when receiving the first hint card. However, after hearing the Nana's idea, Shizue returned to the answer "8/15" and the expression

“2/5÷3/4=8/15” and never wrote other answers. Hearing the Nana’s idea helped Shizue arrive at the conviction that 2/5÷3/4 becomes 8/15.

In the Nana’s idea, the step (iii), 15 small rectangles in the 1m^2 wall, attracted Shizue most strongly. When the teacher and Nana copied her diagram on the blackboard, Shizue looked at them and did not work on her worksheet although her last diagram (Fig. 7) was slightly different from Nana’s (Fig. 8). She started to write new expression when the teacher explained the step (i). But this expression, 2/5÷4/4=2/20, was not related to the Nana’s idea, because “4/4” in it was the number she wrote during her individual working and the way of calculation was the same as her previous way. Even when the teacher mentioned the two vertical lines, she did not react to it. This is consistent with the fact that she did not extend vertical lines when receiving the hint card, whose diagram (Fig. 6) had dotted vertical lines crossing the square. What Shizue directed her attention to was the information that there were 15 rectangles in 1m^2.

The reason why Shizue reacted to this information may be that it could bridge the gap between her cross-multiplying method and the diagram in her worksheet. She had found that there were 8 rectangles in the area painted by 1$d\ell$. Nana’s information about 15 rectangles could complement this finding and validate her initial answer “8/15.” Drawing vertical lines crossing the square might make sense to Shizue as far as it produced 15 rectangles in the square. In other words, it was her findings about the situation that made Shizue react to certain information of the other’s explanation.

It should be noted here that Shizue had drawn the vertical lines before receiving the first hint card (Fig. 5). She had also recognized the small rectangle as the unit for measuring the area painted by 1$d\ell$ of paint. However, Shizue treated these vertical lines and the small rectangle within the larger rectangle in Fig. 7. This can be supported by the fact that she answered “1/4” and “4/4=1” to the questions of the second hint card. The explanation of Nana’s idea made it possible for Shizue to place them in a new context, the square representing the 1m^2 wall. In this sense, the Nana’s idea was not a brand new one, but it placed Shizue’s idea in a new context. Putting her idea in such a new context was, however, critical for linking her calculation method and the findings she noticed during the individual working period.

CONCLUSIONS

The learning processes of the two students have some common characteristics. First, the students selectively picked up the information from others’ explanations. They seemed to select it for bridging the gaps they might feel during their individual workings. Second, using the selected information, the students put what they had known before into the contexts which were different from ones they had adopted during their individual workings. While the students had noticed the basic elements of the ideas before they heard others’ explanations, the selected information enabled them to put those elements in new contexts. And putting them in such contexts resolved the gaps the students felt before that. That is, what the students did when hearing others explaining was to induce their own learning by putting what they had noticed into new contexts activated by the information which they had selected from the others' explanations.

This observation concerning what the students did when hearing others explaining, implies other important points. First, elementary school students who seem to merely hear others' explanations can learn mathematics actively. Second, what students do during their individual workings is a critical factor for such active learning, because it partly constitutes the above-mentioned gaps (*cf*. Nunokawa, 2001) and because such learning can occur by putting it in new contexts. If we remember that the students mentioned in this paper selected the information which they thought could resolve the gaps they felt, it can be said that what students do during their individual workings is also critical for selecting required information from others' explanations. While the information selected from others' explanations brought in new contexts, what the students had done before directed what to be selected and how to use it.

Observing the students' behaviors and learning throughout the lessons, we can say that what students do during their individual workings plays a central role for their learning when hearing others explaining. If we follow this discussion, it may be important for us to encourage students to: (i) try to make sense of problem situations as far as possible so that they can have something to be put in new contexts; (ii) be aware of kinds of gaps between what they are doing, on the one hand, and goals to be achieved or what they know, on the other hand. Such encouragement may generate fruitful time of students hearing others' explanations.

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